TMATH 308 QUINN AUTUMN 2021

Oral Final

Preparation

Use any resources available to prepare including your classmates, the TLC, internet, or me. You should plan, discuss, and debate answers with anyone that is willing to engage. **You are required to cite your sources and collaborators.**

- Sign up for a 25 minute interview slot at `https://calendly.com/jjquinn/oral-final-interview-autumn2021` at your earliest convenience.
- The assessment is **closed book.** You are allowed one side of a 3×5 card for handwritten notes.
- Supporting materials as explicitly mentioned in questions # 1 and # 6 are allowed.
- INTERVIEWS ARE IN PERSON in the TLC. Same place as last time. Please arrive 5 minutes early and get your materials prepared so that we can start promptly. You will have a maximum of 25 minutes to present your solutions.

Grading Rubric

The exam consists of two questions (worth 20 points each) from the next page and oen mystery problem where you will be provided with a matrix, its reduced row echelon form, and asked to describe its related subspaces (worth 10 points). Questions will be graded according to the following rubric:

A	95%	Well-executed. Thorough discussion. All points are well supported. Two or fewer minor errors. No nontrivial errors.
B	85 %	Generally well-executed. Several minor errors; or a nontrivial mathematical error that gets corrected when identified.
C	75%	Uncorrected nontrivial error; or several nontrivial errors that get corrected when identified; or error in fundamental understanding. that gets corrected when identified.
D	60%	Error in understanding of fundamental concept that does not get corrected.
0	0	No evidence of preparation or understanding. Did not comply when requested to view contents of screen.

- *(19 points)* The first question will be **your choice.**
- *(19 points)* I choose the second question from the ones remaining.
  - ∗ You may pass on my choice once for a 5 point penalty. If the pass is used, I select another problem.
- *(9.5 points)* A randomly selected matrix and its reduced row echelon form for you to clearly describe its row space, column space, and null space.
- *(2.5 points)* You can earn and additional 2.5 points by completing the assessment reflection after your interview.

Prof. Quinn's Oral Exam Questions

1. Write 5 true/false questions to assess five *different* and *important* concepts throughout the course that you might put on this exam if you were teaching the class. Then explain the answers, explain why you chose these particular questions and what concepts they are testing. *You are allowed to prepare a document containing your typed questions (only the questions) and bring that to the interview.*

2. The numbered statements are disjoint conditions on the number of rows, m, number of columns, n, and rank, r of an arbitrary matrix A. Given a numbered statement which "lettered" statement(s) below MUST ALWAYS be true? Be prepared to fully justify your answer and beware, anywhere from none to all of the lettered statements may match for each numbered one.

   Assume A is an $m \times n$ matrix with rank r.

   (1) $m = n = r$.

   (2) $m = n > r$.

   (3) $m > n = r$.

   (4) $n > m = r$.

   (5) $n > m > r$.

   A. There always exists at least one solution to $A\mathbf{x} = \mathbf{b}$.

   B. Assuming $\mathbf{b}$ is in the column space of A, $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.

   C. Assuming $\mathbf{b}$ is in the column space of A, $A\mathbf{x} = \mathbf{b}$ has an unique solution.

   D. A has an inverse.

   E. The columns of A are linearly independent.

   F. $\det(A) = 0$.

3. There has been lots of vocabulary in this course. It is important to understand the relationship between the definitions by combining words and constructing examples. Be able to give and explain matrix examples that satisfy matched characteristics, one from column A and one from B (or say why one doesn't exist.) (4x)

Column A	Column B
1) consistent	a) diagonalizable
2) inconsistent	b) not diagonalizable
3) singular	c) triangular
4) nonsingular	d) symmetric

4. Given an $n \times n$ matrix A and scalar λ, define a subset of $\mathbb{R}^n$ by

$$S = \{\mathbf{v} \in \mathbb{R}^n \quad | \quad A\mathbf{v} = \lambda\mathbf{v}\}.$$

   Show that S is a subspace.

5. Let A and B be similar matrices. For any scalar $\lambda \in \mathbb{R}$ show that

   (a) $A - \lambda I$ and $B - \lambda I$ are similar, and

   (b) $\det(A - \lambda I) = \det(B - \lambda I)$.

   Why would a mathematician find result (b) interesting?

6. Of the written homework you submitted this quarter where you were asked to "show" something in general, identify the problem that you found most challenging, critique your written submission, and present an improved solution. *Have a copy of the original submission to share and discuss.*

For your final question I will give you a matrix A and its reduced row echelon form $U = \text{rref}(A)$. You will asked to describe the row space of A, the column space of A, and the nullspace of A. I provide you with one example below, you can practice with problems from §3.6.

$$A = \begin{bmatrix} 2 & 2 & 0 & 1 & 2 \\ 1 & 2 & 1 & 2 & 3 \\ 2 & 4 & 7 & 9 & 11 \end{bmatrix} \qquad U = \begin{bmatrix} 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Permutations for problem 3:

C B D A
B C D A
A C B D
D C A B
A D C B
A C B D
C B A D
D C B A
B D A C
B A D C
C A B D
B D C A
B C A D
C D B A

Matrices A and $U = \mathrm{rref}(A)$. Find the row space, column space, and nullspace of A.

1. $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 6 & 4 \\ 1 & 2 & 4 & 3 & 6 & 5 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 2 & 0 & 0 & -5 & 23 \\ 0 & 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \\ 5 & 6 & 11 \\ 6 & 4 & 10 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & -2 & 0 & 3 & -4 \\ 3 & 2 & 8 & 1 & 4 \\ 2 & 3 & 7 & 2 & 3 \\ -1 & 2 & 0 & 4 & -3 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 3 & 2 & -24 \\ -2 & 2 & 3 & 48 \\ 0 & 8 & 7 & 0 \\ 3 & 1 & 2 & 96 \\ -4 & 4 & 3 & -72 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -49 \\ 0 & 0 & 1 & 56 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. $A = \begin{bmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & 4 & -3 & -6 \\ 7 & 14 & -6 & -3 \\ -2 & -4 & 1 & -2 \\ 2 & 4 & -2 & -2 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 7 & -2 & 2 \\ 4 & 14 & -4 & 4 \\ -3 & -6 & 1 & -2 \\ -6 & -3 & -2 & -2 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & 0 & \frac{5}{9} & \frac{2}{9} \\ 0 & 1 & -\frac{4}{9} & \frac{2}{9} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$