Remote Oral Final Questions

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1. Write 5 true/false questions that illustrate a variety of ideas from this course that you might put on this exam if you were teaching the class. Then explain the answers, explain why you chose these particular questions and what *Type your questions and bring them with you.*

2. Of the written homework you submitted this quarter, identify the problem that you found most challenging, critique your written submission, and present an improved solution. *Have a copy of the original submission to share and discuss.*

3. Consider one mathematical idea from the course that you have found beautiful, and explain why it is beautiful to you. Your answer should: (1) explain the idea in a way that could be understood by a classmate who is familiar with vectors and matrices but has not yet taken this class and (2) address how this beauty is similar to or different from other kinds of beauty that human beings encounter.

4. Discuss the parallels between arithmetic in Matrix Algebra and arithmetic that you learned in secondary school. In particular, what strategies for solving algebraic equations work the same way for matrix equations and what do not? Be sure to illustrate your discussion with examples of both.

5. Choose one interesting proof problem from the text that was not assigned. Describe why you find it interesting. Then either solve it or find a solution online and work through it, using your own understanding to critique that solution and improve it (be sure to cite your sources).

6. Read Section 5.2 and learn about *orthogonal subspaces* and *orthogonal complements.* Digest, understand, and be able to explain Theorem 5.2.1, the Fundamental Subspace Theorem, and its relationship to Theorem 3.6.5, the Rank-Nullity Theorem.

7. Is there a relationship between nonsingular matrices and diagonalizable matrices? Is it possible to have a matrix that is both nonsingular and diagonalizable? Nonsingular and not diagonalizable? Singular and diagonalizable? Singular and not diagonalizable? Be sure to give illustrations (and make sure your examples are your own, that is to say I don't want everyone bringing me the same examples to illustrate existence.)

8. The linear operator of $\mathbb{R}^2$ that rotates every vector by an angle θ in a counterclockwise direction is represented by the matrix
$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$
Find the eigenvalues and eigenvectors associated with A and interpret their meaning geometrically.

9. The numbered statements are disjoint conditions on the number of rows, m, number of columns, n, and rank, r of an arbitrary matrix A. Given a numbered statement which "lettered" statement(s) below MUST ALWAYS be true? Be prepared to fully justify your answer and beware, anywhere from none to all of the lettered statements may match for each numbered one.

Assume A is an $m \times n$ matrix with rank r.

(1) $m = n = r$.
(2) $m = n > r$.
(3) $m > n = r$.
(4) $n > m = r$.
(5) $n > m > r$.

A. There always exists at least one solution to $A\mathbf{x} = \mathbf{b}$.
B. Assuming $\mathbf{b}$ is in the column space of A, $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.
C. Assuming $\mathbf{b}$ is in the column space of A, $A\mathbf{x} = \mathbf{b}$ has an unique solution.
D. A has an inverse.
E. The columns of A are linearly independent.